CASE STUDIES OF REGRESSION AND MARKOV CHAIN MODELS

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ABSTRACT

There are two main types of models available for modelling pavement performance behaviour. They are regression type models and models based on Markov Chains. Regression type models are the easiest to use and allow for the analysis of various factors. The advantages of Markov Models are that they can be calculated with a minimum of two years of data unlike regression models which require data over a period of years to predict trends. In addition, Markov Models permit the use of expert opinion or a Bayesian approach in the development of performance curves.

This paper looks at three case studies of performance models for pavements and Bituminous Surface Treatments (BSTs) in a remote area where there were no available performance models. The initial models were based on two years of data from 1990. These models are compared with the actual performance of the pavements and BSTs. In addition, the Markov models have been recalculated with 6, 9, 13, 19 and 23 years of data and the models are compared to the original model and actual performance over time.

The analysis of the comparative data indicates that in most instances the Markov models underestimated pavement performance.

INTRODUCTION

Regression models have long been used to model pavement performance. The advantages of regression analysis techniques are but not limited to:

- Ease of computation.
- Future trends can be predicted based on historical data.
- It provides a statistical platform for more advanced modeling.
- It has the ability to deal with multivariate components.
- It allows for an evaluation of the “fit” of the model prediction and the data.

Regression models are a statistical tool for investigating correlations/relationships between variables. Usually, the model seeks to ascertain the causal effect of one variable upon another. The effects of a person’s age on bone density or in terms of pavements, the effect of changes in asphalt grade on thermal cracking are an example of that variable relationship. The model assembles data on the underlying variables of interest and employs regression to estimate the quantitative effect of the causal variables upon the primary variable that they influence. Regression analysis can also assess the “statistical significance” of the estimated relationships, that is, the degree of confidence that the true relationship is close to the estimated relationship.

Markov Chains (*1*) have been used over the years for statistical analyses of engineering, medical and environmental data. Markov Chains have been used in transportation related fields including air traffic control (*2*), bridge deterioration rates (*3*), intersection queuing theory (*4*), flexible pavement crack deterioration (*5*) and pavement performance (*6*), (*7*), (*8), (9), (10*).

The advantages of Markov Chain modeling include:

- Ability to develop a prediction model with just two years of data, unlike regression models which need data over a period of years to predict trends.
- Ability to calculate performance even if data for some years is missing.
- Ability to treat data with stochastic tools including Bayesian processes.
- Avoiding tedious regression techniques in the presence of a multitude of tributary factors.
- Avoiding the "survival of the fittest phenomenon". When data from older pavements based on the average of "x" year old pavements are calculated, the performance of the pavement appears to improve. As poorer pavements are rehabilitated, the resulting average of the remaining pavements increases as only the best performing pavements, "the fittest", are included in the "x" year average. This distorts "state in time" (curves fitted to the actual data) and regression type models

A proper analogy of Markov chains would be the game of Monopoly. What matters is the current position of the token on the board. The next position (or state) of the token depends only on the current position, and the next roll of the dice. In the case of pavements, what matters is its current state and where it ends up after the next duty cycle (normally a year) based on the probability of similar pavements after one cycle. The establishment of a Markov Chain is a snapshot of a system and a snapshot one cycle later to establish the transitional probabilities over that cycle. To establish a comparable performance curve using regression analysis, a number of successive years of data are required.

In the late 1980s, pavement management systems for asphalt concrete and BST (Bituminous Surface Treatment) were developed for the highway systems in northern British Columbia and Yukon. These highways were in the northern part of Canada and it was difficult to obtain similar pavement performance data for facilities with such extreme climatic parameters. Markov models were an obvious choice to develop initial performance models as only two years of data (current state + one duty cycle) were needed (*11*). Now that there are 22 years of performance data, it is interesting to compare the predictions of the initial model and Markov updates over that period.

PAVEMENT CONDITION INDEX (PCI)

At the network level, a generalized model of pavement performance permits the prioritization and optimization of strategies needed for long-term budgeting and planning. The network analysis uses an overall condition rating for selecting rehabilitation priorities, and for investigating the impact of varying funding levels.

The PCI (*12*) is such a single index that combines all the distress extent and severity factors with ride score for pavements. This index is particularly useful in the network phase of a pavement management system. The PCI is an overall rating of the pavement section on the scale of 0 to 100. The higher the determined PCI index is, the better the condition of the pavement.

The PCI is calculated as an index based on the severity and extent of durability distresses (ravelling, bleeding, and rippling), structural distresses (rutting and wheel track cracking) environmental/load (transverse, edge, centreline, meander, block and random cracking), distortions, and surface roughness (ridescore or International Roughness Index (IRI)).

In general terms, based on actual performance ratings, a highway with a PCI of 73 or greater is in very good condition, a highway with a PCI of 68 to 73 is in good condition, highways with a PCI of 60 to 68 are in fair condition, a highway with a PCI between 55 and 60 (poor) requires an overlay/milling-overlay and highways with PCI's less than 55 are in very poor condition and need extensive repairs before resurfacing. Figures 1 to 5 show typical pavement PCI conditions.

Bituminous Condition Index (BCI)

The Bituminous Condition Index (*13*) is a similar composite value for BST. BSTs are used extensively in Northern Canada as an alternative to pavement where traffic volumes are lower. BSTs consist of a single application of well-graded aggregate applied directly to an asphalt film sprayed on the subgrade or base course. A BST has no structural value. It can be applied directly on the subgrade as a dust palliative or on a full pavement structure of base and subbase with a BST surface instead of asphalt concrete. When traffic volumes warrant and budgets permit, the BST is replaced with asphalt concrete.

The BCI is calculated as a index based on the severity and extent of durability distresses (ravelling and bleeding), structural distresses (rutting, subgrade failures, and shoulder disintegration) environmental/load (cracking, potholes) distortions, patching, construction deficiencies (joints, streaking, and corrugations), and surface roughness (ridescore or IRI)

The BCI is an overall rating of the pavement section on the scale of 0 to 100. The higher the BCI value, the better the condition of the pavement. Figures 6 to 10 illustrate varying BCI level conditions.

In general terms, a highway with a BCI of 75 or greater is in very good condition, a highway with a BCI of 65 to 75 is in good condition, a highway with a BCI of 60 to 65 is in fair condition and is in need of rehabilitation, a highway with a BCI between 55 and 60 (poor) needs to be reshaped with some additional strengthening and a highway with BCIs less than 55 is considered to be in very poor condition and needs extensive repairs before rehabilitation.

Regression Analysis Considerations

The goal of a regression analysis is to determine the values of the main parameters that cause a function to best fit a set of existing data observations. The regression analysis uses data containing the values of the dependent and independent variables for a set of observations. The more observations you provide, the more accurate the estimate of the parameters will be. In this study the regression analysis compares the relationship between age and performance (PCI/BCI). Given that all pavements and BSTs were designed using the same design methods, age is a good proxy to the extent that the design systems properly accounted for subgrade strength and traffic volumes.

If a perfect fit existed between the function and the actual data, the actual value would exactly equal the predicted value. This is very rarely the case. The difference between the actual value of the dependent variable and its predicted value is the error of the estimate which is known as the residual error. The aim of regression analysis is to minimize the sum of the squared residual values for the set of observations. This is the least squares (r^2) regression fit method where r^2 is a measure of the fit. An r^2 of 1.0 demonstrates a perfect correlation between variables and an r^2 of 0.0 means that there is no correlation between the variables.

Linear, squared, cubic, exponential and periodic regressions were fitted to the data. The cubic model gave the highest r^2 or the best fit for both the pavements and BSTs in this study. The generalized form of the equation is:

$$PCI = (a) - (b) \times (AGE) + (c) \times (AGE^2) - (d) \times (AGE^3)$$

Where a, b, c and d are regression constants.

Bayesian Regression and Markov Transition Matrices Considerations

Bayesian regression analysis has been used as the tool for performance modeling (*6*) (*11*) (*14*). It involves a technique where prior knowledge and experience are combined with data from field observations, as shown schematically in Figure 11.

One of the main techniques used to model Bayesian analyses are Markov Chains. As long as there are pavements in a variety of states, the progression model can be developed independently of age. Markov Chains as predictive models, also capture the uncertain behaviour of pavement deterioration using statistical principles (*6*).

A pavement begins its life in a nearly perfect condition and then is subjected to a sequence of "duty cycles" that cause its condition to deteriorate (*11*). For the purposes of this study, a duty cycle is defined as one full cyclic year of weather and traffic.

A State Vector indicates the probability of a pavement section being in a given state in any given year. The Initial State Vector is the State Vector at time zero or immediately after construction. For example (Table 1), an initial state vector could indicate that there is a 31 percent chance that a new pavement would have an initial PCI between 82.5 and 77.5 (nominal 80 PCI), a 29 percent probability that it would have a PCI between 77.5 and 72.5, etc. The average PCI at time zero is found by taking the weighted average for the probabilities:

Average PCI at Year (t=0) = 80 x 0.31 + 75 x 0.29 + 70 x 0.17 etc.

The transition matrix is created by comparing data for the same section for two consecutive years and determining the probability that a pavement in condition "y", would remain at the same state or decrease to a different level after a duty cycle. For example, (Table 1) a pavement that has a PCI rating between 82.5 and 77.5 in year x has a 20 percent chance of remaining in that state one year later, a 40 percent chance of falling a level to between 77.5 and 72.5 and a 40 % chance of falling into the 72.5 to 67.5 category in year “x”+1.

Initially, the table can be filled using one of two techniques. The first is to use a Bayesian approach where a group of experts is asked to fill in the matrix based on their experience. This has the advantage of developing a model with very little data. The second and preferred approach uses actual data. However, if any transition levels are missing, the Bayesian approach is used to fill in the gaps. For the Markov Matrix to work there has to be an entry level at each state i.e. there cannot be any gaps in the data. As more data becomes available the model can be updated to provide greater accuracy in predictions.

The State Vector v (t) for any age “t” of a pavement can found by multiplying the initial state vector v (0) by the transition matrix raised to the power “t”. So:

State Vector in year 0	v (0)
State Vector in year 1	v (1) = v (0) x matrix
State Vector in year 2	v (2) = v (0) x matrix^2
State Vector in year 3	v (3) = v (0) x matrix^3

BASE ASSUMPTIONS FOR MARKOV MODELING

There are a number of inherent assumptions that must be considered in the modeling of pavement and BST performance using Markov techniques. Among the assumptions are:

- all pavements have been designed using a pavement design method that accounted for subgrade strength and traffic volumes. In other words, variations in traffic and subgrade soils have been accounted for in the design of the pavement structure and all pavements are designed to give a common service life.
- all pavements receive a common level of maintenance.
- all pavements are in a common climatic zone.
- all pavements have been rated using a common rating system and the data verified with a Quality Assurance/Quality Control (QA/QC) system.

COMPARISON OF MARKOV PREDICTIONS FOR PAVEMENTS

Markov prediction models for statistically forecasting changes of PCI over time were initially made with three years of data in 1990 and with nine years of data in 1996. These initial Markov prediction curves have been supplemented in this paper based on 13, 19 and 23 years of data to study the changes in the prediction model and to compare the Markov curves to actual pavement performance.

On an individual section by section basis, Figure 12 indicates that the actual data corresponds to the regression analysis rather than the Markov model. As the correlation between the data and the regression models was very good, the terms “data” and “regression model” are used interchangeably in this paper.

Figure 13 indicates the Markov prediction of the performance of northern pavements based on 3, 9, 13, 19 and 23 years of data. Also shown in the figure are regression curves fitted to the actual field data. Figure 13 indicates that the Markov models underestimated the performance of the pavements consistently. One method of evaluating the relative importance of the underestimation is the comparison of pavement service life.

Trigger values are used to establish PCI levels at which various types of pavement rehabilitation are required. During the implementation of the pavement management system, the rating panel suggested an initial recommendation for maintenance or rehabilitation. The intent of this was to serve as a quality assurance check on the ratings – normally a pavement with few distresses would not be recommended for an overlay. If the rating form indicated rehabilitation, the entire rating would be rechecked. As a by-product, the comparison of the calculated PCI with the rehabilitation strategy provided a reliable method of establishing trigger values.

A study of this data (*16*) indicated that the panel recommended adding an overlay when the pavement had a PCI of 60 to 63. Between a PCI of 55 and 60 a milling operation or an additional thicknesses of pavement is required to restore the pavement cross-section. Below a PCI of 55, some form of subgrade and base rehabilitation was required.

As indicated in Table 2, the use of a Markov Model consistently under-estimated the service life of the pavements by 50 percent even with the addition of over 800 transitions. The Figures and Table also indicate that there is little significant change to the Markov model even after the addition of 9 years of data.

COMPARISON OF MARKOV PREDICTIONS FOR BST

When the BST Management System was implemented, the system was mainly BST directly on a subgrade or a nominal amount of base course. Over the years the system has been converted to BST on full base and subbase courses. This paper deals only with BST on base and subbase courses. The initial Markov model was based on a large geographical area to obtain a sufficient number of points to formulate a Markov Chain. With the passage of time, it became evident that there were variances in behaviour of different sections of the Alaska Highway. The data shown here are for two geographic areas with different traffic levels. The highway between kilometre 133 and 550 had traffic in the 1000 vehicles per day (vpd) while the highway between km 550 and 950 had traffic volumes in the 500 vpd range. The expected lives shown in Table 3 for the 2 and 7 year Markov models were based on data from all sections of the highway including these two zones. The expected lives from year 11 to year 22 are based on these two distinct geographical areas only.

Figure 14 indicates that for both the regression models and the Markov models, the performance curves were stable for the km 133 to km 550 sections. By contrast, Figure 15 shows that both the regression model and the Markov Model in particular are relatively unstable after 5 years of service for the sections between km 550 and 950.

The trigger value for the rehabilitation of BSTs is a BCI between 63 and 65. Below a BCI of 55 generally means that a major rehabilitation or strengthening of the highway is required. Very few sections reach values of 55 and this fact has influenced the shape of both the regression and Markov performance curves.

Table 3 indicates that for the sections between km 133 and km 550 the Markov model underestimates the service life at both intervention levels (63 and 65) by 30 to 40 percent. At the BCI=55 or reconstruct level, the regression models indicate service lives of 14 to 15 years while the Markov models indicate service lives in excess of 16 years.

For the section between km 550 and 950, the regression models give more consistent service life predictions at the 65 and 63 intervention levels (14 and 13 years respectively) than the Markov model. Both models predicted that it would be more than 15 years before the BCI would fall below 55 if there were no rehabilitation activities before this date.

There were no readily available explanations for the difference between the regression and Markov models. The only way that a better agreement could be obtained was by removing significant amounts of data particularly in the 0 to 5 year range from the Markov model.

SUMMARY AND CONCLUSIONS

This paper investigated the use of Markov and Regression models for the determination of suitable performance models of asphalt concrete pavements and BST in a northern climate where no other models were available. Markov models have the advantage of giving pavement prediction curves with only a few years of data. For pavements the Markov models gave consistent results with 100 transitions. Markov and Regression models both gave consistent performance curves as more data was added. However, the Markov Model underestimated actual pavement service life by 40 to 50 percent.

Initial BST performance curves were based over a large geographical area to obtain sufficient data. With the addition of newly constructed BST sections, it became obvious that the BST was performing differently in different sections. Two areas of BST were selected for further study, one with traffic volumes of 500 vpd and the other with traffic of 1000 vpd. For the 1000 vpd sections, both the Markov and Regression models gave consistent results with the Markov model underestimating the service life by 30 to 40 percent. For the 500 vpd sections the regression models were more stable than the Markov models. The Markov Models also underestimated the service lives of these sections.

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TABLE 1 Initial State Vector and Transition Matrix

					Initial State Vector					
N	PCI	80	75	70	65	60	55	50	45	40
35	Probability	0.312	0.286	0.171	0.116	0.115	0.000	0.000	0.000	0.000
					Transition Matrix					
N	PCI/yr	x 80	x+1 75	x+2 70	x+3 65	x+4 60	x+5 55	x+6 50	x+7 45	x+8 40
5	80	0.200	0.400	0.400						
15	75		0.185	0.126	0.438	0.251				
29	70		0.035	0.241	0.448	0.172	0.069	0.035		
84	65				0.345	0.500	0.131	0.024		
102	60				0.098	0.431	0.274	0.147	0.029	0.021
72	55					0.153	0.528	0.236	0.083	
40	50						0.079	0.484	0.256	0.181
28	45							0.143	0.607	0.250
6	40								0.167	0.833

TABLE 2 Comparison of Markov Prediction of PCI Service Life versus Actual Performance

PCI		Service life					
Years of Data	Count (N)	Trigger Value: 63		Trigger Value: 60		Trigger Value: 55	
		Markov	Regression	Markov	Regression	Markov	Regression
3	51	6.5	-	7.5	-	9.5	-
6	118	6	-	7	-	9.5	-
9	339	5.5	9	7	12	10.5	19
13	465	5.5	9.5	6.5	15	10	>25
19	739	5.5	9	7	15	9.5	22
23	803	5.5	10	7	15	11	20

TABLE 3 Comparison of Markov Prediction of BST Service Life versus Actual Performance

BCI	Years of Data	Count (N)	Service life: BCI Trigger Value: 65 Markov	Regression	BCI Trigger Value: 63 Markov	Regression	BCI Trigger Value: 55 Markov	Regression
BST Km 133 to Km 550	2*	175*	4	-	5	-	9	-
	7*	702*	8	-	10	-	-	-
	11	342	3.5	5	6.5	11	>16	15
	16	626	3.5	-	6	-	>16	-
	17	484	3.5	5.5	6	11.5	>16	14.5
	19	598	3.5	5	5.5	10.5	>16	14.5
	22	859	3.5	6	6.5	9.5	>16	12.5
BST Km 550 to Km 950	2*	175*	4	-	5	-	9	-
	7*	702*	8	-	10	-	-	-
	11	87	8.5	13.5	10	>16	15	>16
	16	225	>16	>16	>16	>16	>16	>16
	19	328	15	11	>16	12	>16	14.5
	22	426	9.5	12	14	14	>16	>16

**Data from km 133 to km 1635*

Figure 1 Pavement with a PCI 78

Figure 2 Pavement with a PCI of 71

Figure 3 Pavement with a PCI of 64

Figure 4 Pavement with a PCI of 58

Figure 5 Pavement with a PCI of 43

Figure 6 BST with a BCI of 79

Figure 7 BST with a BCI of 71

Figure 8 BST with a BCI of 63

Figure 9 BST with a BCI of 48

Figure 10 BST with a BCI of 40

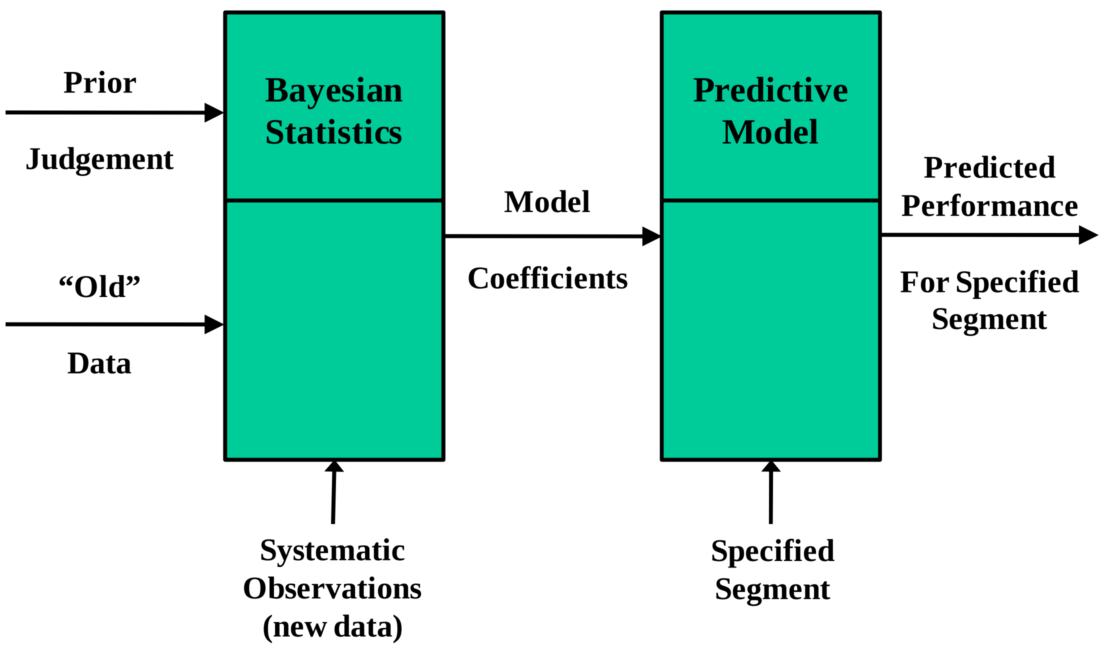

Figure 11 Bayesian Statistical Approach for Combining Prior Knowledge - Experience with Data (*15*).

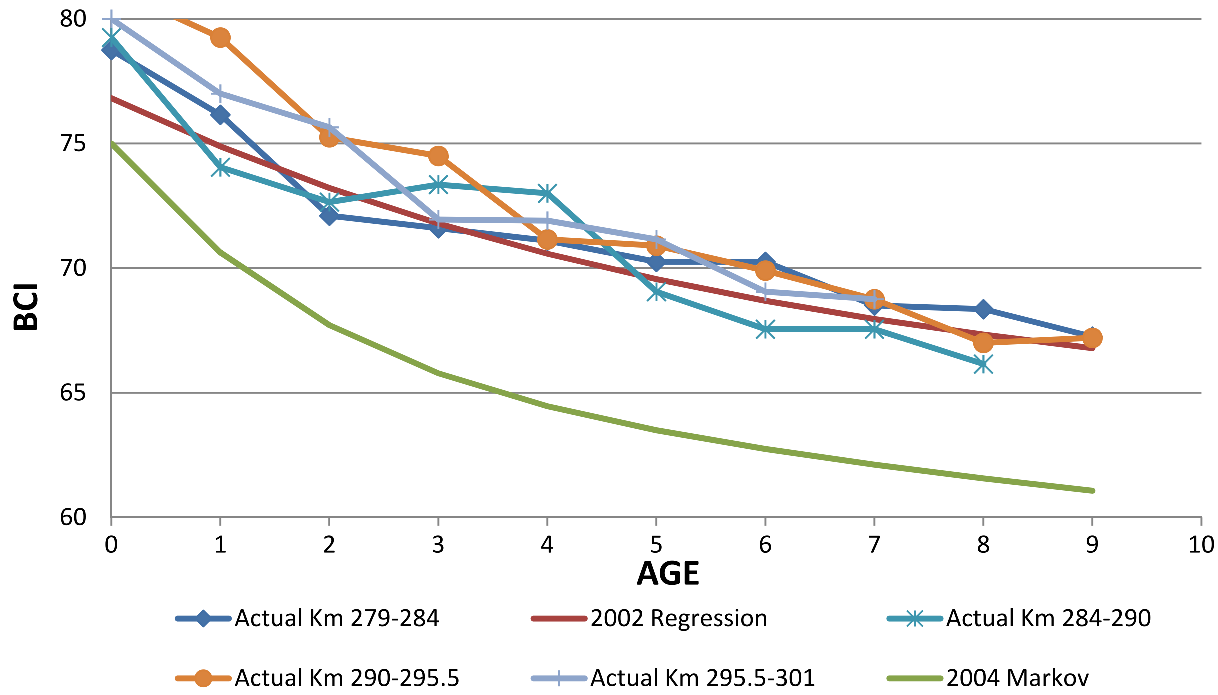

Figure 12 Comparison of Data and Regression and Markov Models

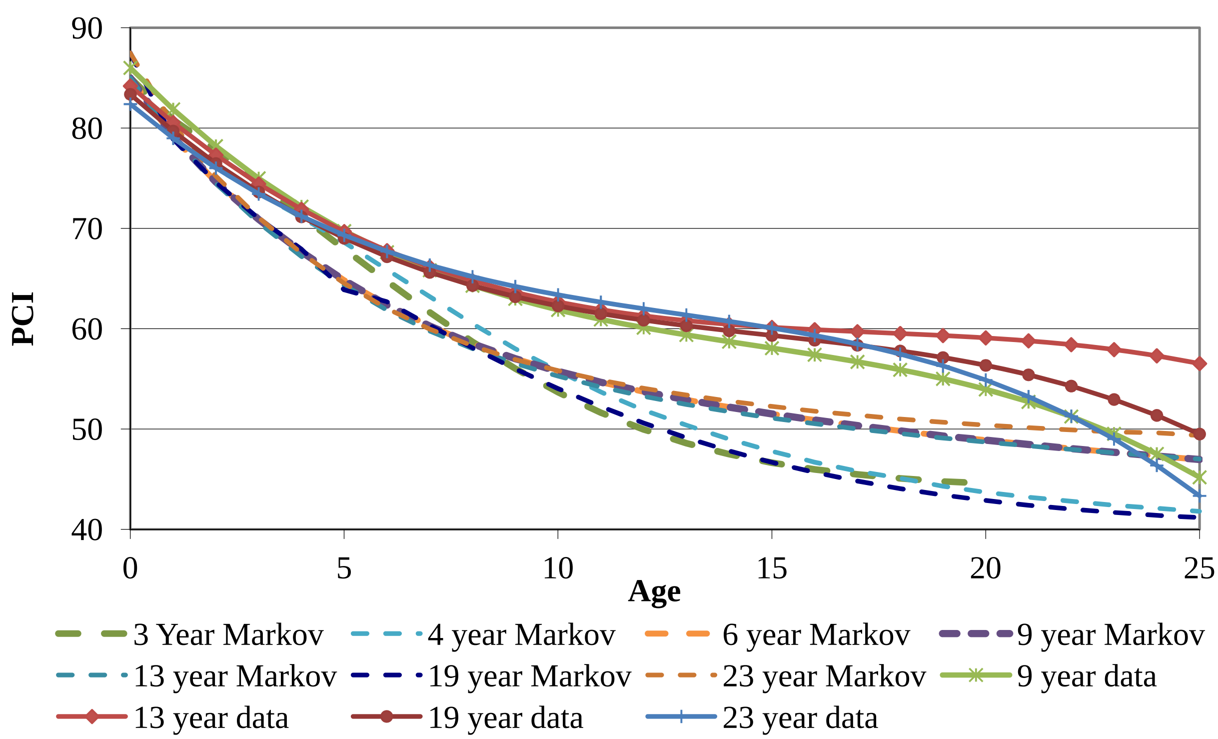

Figure 13 Pavement Performance Curves Based on Markov Predictions and Actual Performance

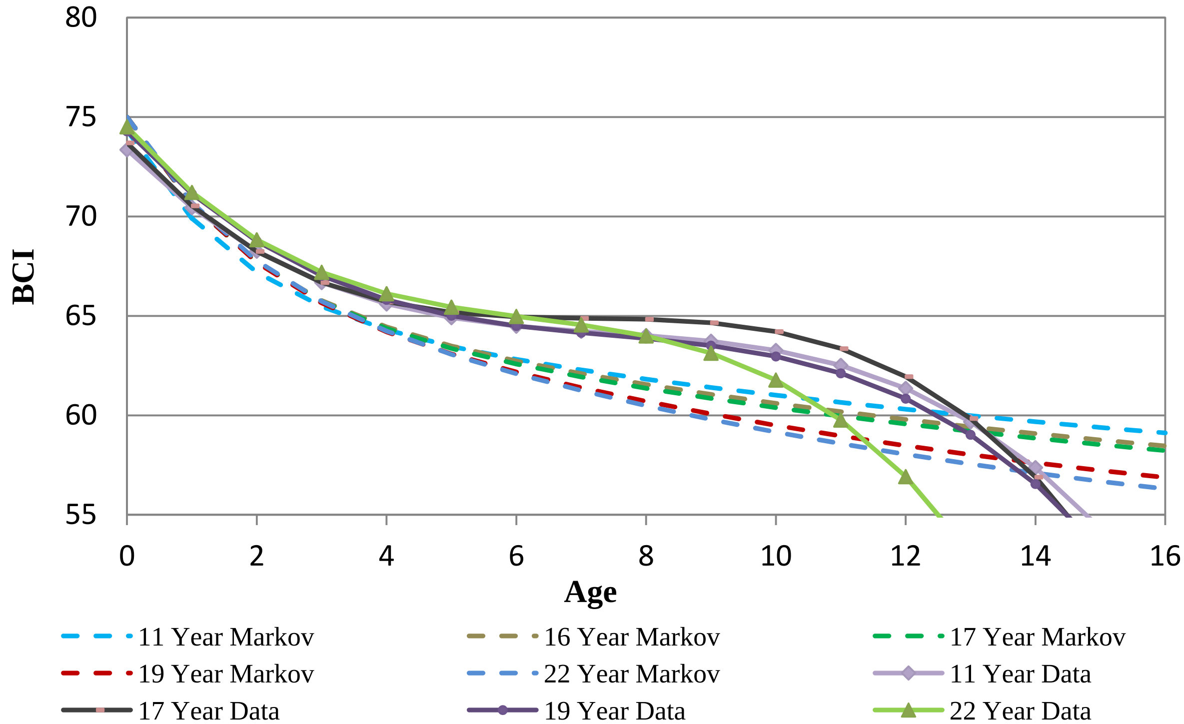

Figure 14 BST Km 133-550 Performance Curves Based on Markov Predictions and Actual Performance.

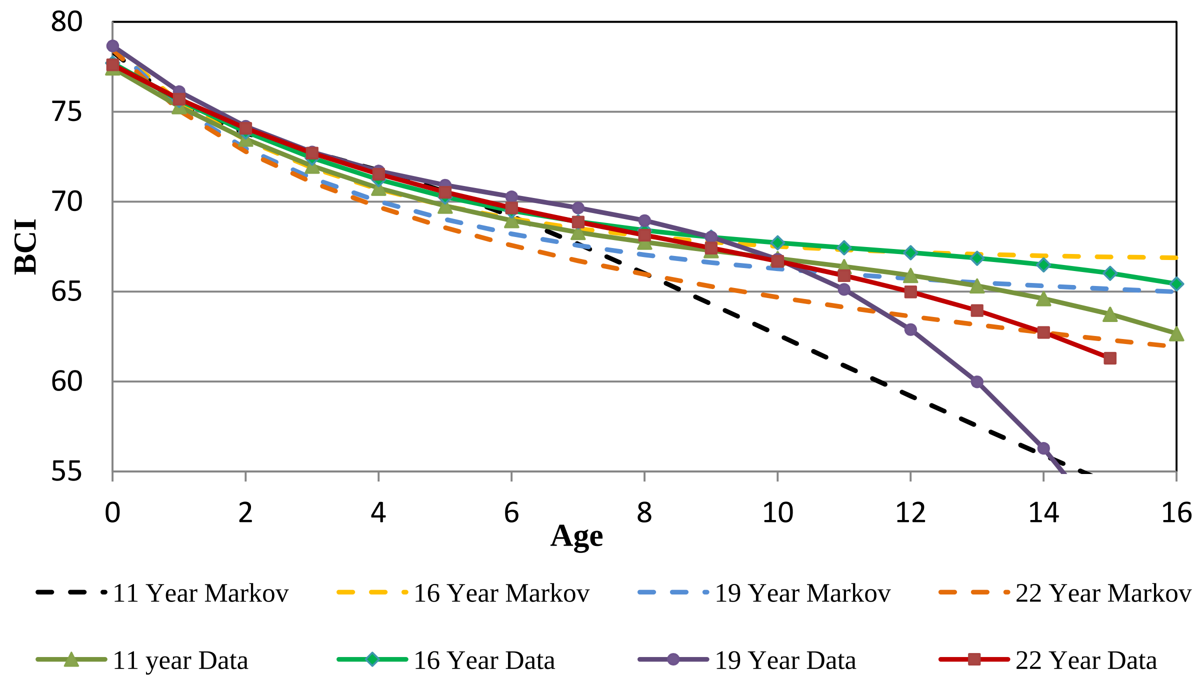

Figure 15 BST Km 550-950 Performance Curves Based on Markov Predictions and Actual Performance.