

BCT Pilot Project for Culvert Integrity Analysis in the City of Toronto

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Abstract

Infrastructure owners must decide between rehabilitating or replacing aging assets, however, appropriate asset health information is simply not available for certain structures. An asset class identified by The Ontario Good Roads Association (OGRA) that presently lacks health information is major roadway pipes (culverts).

A critical culvert failure mode is due to the formation of undermining or voids behind the culvert wall caused by soil erosion. The level of “undermining” is currently not measured when evaluating replacement or rehabilitation possibilities. This information gap is of particular concern for planning “slip-lining” or other rehabilitation projects since significant undermining compromises the culvert’s fundamental structural integrity. Lack of complete integrity information can lead to unnecessary and costly replacements, and poses a risk to public safety.

During the summer of 2011, OGRA supported an integrity analysis pilot project on selected roadway culverts using recently-developed inspection equipment capable of quantifying undermining. This new modality, Backscatter Computed Tomography (BCT), is a practical *in-situ* form of Computed Tomography also known as a CAT Scan. The purpose of the pilot was to evaluate the technology’s potential to measure undermining, thereby facilitating the planning of major roadway culvert replacements or rehabilitation. Three culverts in the City of Toronto were inspected to determine the culverts’ structural condition using BCT equipment provided by Inversa Systems Ltd.

Introduction

North America invests significant resources in maintaining its civil infrastructure. Time and environmental conditions wear away the supporting soils surrounding pipes. An improperly supported pipe will experience mechanical stresses that will eventually result in complete structural failure, which can collapse the road above. This lack of supporting material creates challenges for planning repairs or rehabilitation, as key asset integrity information is unavailable. Optimizing these decisions can yield significant cost savings for asset owners; part of this optimization process is meaningful asset integrity diagnostic information.

The concept of deploying Backscatter Computed Tomography (BCT) for pipes emerged from discussions between New Brunswick Department of Transportation (NBDOT) engineering staff and Inversa Systems, a provider of BCT equipment. The result was an explorative feasibility study, including a trial inspection of a test pipe. This study was completed in December 2009 and demonstrated the utility of BCT in identifying undermining. In the fall of 2010, NBDOT conducted a pilot project consisting of four culverts to further understand BCT’s potential in assessing undermining beneath large diameter or deeply buried pipes.

Learning of this capability, OGRA facilitated a BCT demonstration for the City of Toronto. The city’s municipal engineers subsequently selected six potential culverts, of which three were selected for BCT inspection.

Backscatter Computed Tomography

BCT inspects a structure by examining it with a beam of gamma-rays and measuring the backscatter radiation. Tomographic imaging is conventionally based on transmission measurements that require access to opposing sides of the inspected object. A similar cross-sectional image to that of medical CT (CAT scan) is produced by BCT. However, by employing backscattered radiation, BCT has the unique ability to inspect infrastructure of almost any material using a single side of access.

The first scatter system was developed by Lale [1] in the late fifties. Proposed as a medical scanner, the system consisted of a well collimated detector facing a source beam. This layout is shown in Figure 1(a). Since the detector only received photons scattered from a specific location in the object, the detector directly provided an indication of the electron density of that location. A scan was conducted by moving the target in a rectilinear fashion until a signal was recorded for the desired portions of the object. Clark and Van Dyk [2] developed a similar system. The physical layout was slightly different; the scatter detector was located perpendicular as opposed to directly in line with the x-ray beam (Figure 1(b)). Farmer and Collins [3] designed a system that used a mono energetic γ -ray pencil beam, but the detector had a wide field view and discriminated energy. Energy information was then used to determine the location along the path of the incident beam where scattering occurred.

Later systems employed more advance software and calculated attenuation during image reconstruction. Battista and Bronskill [4] developed a method to determine attenuation coefficient using a scanner configuration similar to Clark and Van Dyk [2]. The electron density of a corner voxel, located close to the detector, is first measured. This known electron density value is then used to determine the attenuation coefficient of the adjacent voxel. The electron density of the adjoining voxel is then calculated using the known attenuation coefficient to correct for attenuation. This pattern is repeated until the complete image is reconstructed.

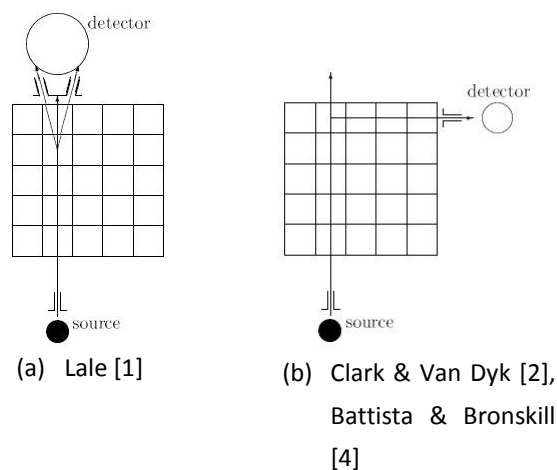


Figure 1: Layouts of various reconstructive scatter imaging scanners

A more recent technique extended the solution's linear domain to accommodate thick/dense structures, but requires measuring scattering from the structure one strip at time [5]. A theoretical method devised by Arsenault [6] at the Laboratory for Threat Material Detection (LTMD) at the University of New Brunswick in 2004 overcomes challenges for dense materials used in infrastructure without the use of transmission measurements or strip-by-strip scanning. The commercial BCT equipment used for the demonstrations is based on this research from the LTMD.

In the case of roadway infrastructure, the BCT tool provides "snap-shots" through the culvert wall into the surrounding backing material. Solid backing or undermining is easily discerned through diagnostic images. In the case of "undermining", water flowing from a stream or through the supporting soil above can erode a pathway through the soil surrounding the pipe. In some instances this may be visualized on the terminal ends of the pipes as a "rat hole"; with the depth or extent of the damage under the pipe unknown.

Culvert Inspection Protocol

BCT is suitable as the final step in the inspection protocol, following visual and acoustic inspection, in deciding whether to repair or replace a structure. The trial employs a comprehensive inspection protocol with BCT to quantify suspected undermining.

Inspection Protocol

Currently, visual inspections are primarily used to assess the condition of questionable pipes. During examination, the inspector uses a number of visual indicators to determine a structure's operational viability. However, certain internal defects, not visible from visual inspection alone, can cause catastrophic pipe failure. Some visual indicators are sufficiently obvious (vegetation, overgrowth, slope and pavement conditions) making further inspection unnecessary. However in other cases, such as "undermining", visual indication does not provide useful information.

The methodology employed involves a three-part inspection to assess the culvert's structural integrity. This consists of conducting a visual inspection, an acoustic inspection and BCT imaging.

Visual Inspection – the visual inspection assesses eleven features:

- Pavement
- Guardrail
- Embankment
- Channel Alignment
- Bank Protection
- Waterway Blockage
- Scour
- Abutment
- Deterioration
- Cracks, Seams and Joints
- Shape

Each feature is coded 0 through 9 with 9 being "as built" and 0 being "complete failure". Each code has a specific feature description.

Acoustic Inspection – the purpose of the acoustic inspection or "knock test" is to determine the extent (quantity not magnitude) of suspected voids within the pipe and aid in prioritizing which anomalies can be imaged. A detailed knock test was conducted at two corrugation increments (approximately every 15-

20cm) on both sides of the pipe at five relative elevations: waterline, pipe midline, forty five degrees above and below midline and roofline. Based on the acoustic signal, a map of the suspected voids is created and imaging is undertaken at the worst case locations first. **[Note:** The first 3.0m at the terminal ends of the culvert are considered transition zones and not mapped unless voids are visible or extend beyond the 3.0m.]

It is important to note that the acoustic inspection cannot tell the true extent of the damage, only that a void may be present. The indication of a void may relate to a small 1cm void limited to a single corrugation or could indicate a deep void spanning many corrugations. No current procedure governs where or how to do an acoustic inspection, how to interpret them, or how to capture the information for future deterioration trending. Furthermore, acoustic inspection is subjective and does not provide quantifiable information. However, it is a fast and inexpensive way to identify suspected undermining.

An acoustic inspection was performed as per the Inspection Protocol section of this report. Acoustic anomalies are approximated as either rectangles or lines depending on their size. Anomalies spanning 1 to 3 corrugation widths or those which are less than 30 vertical centimeters, are approximated as lines; anomalies larger than this are displayed as rectangles. A distance measurement along the length of the culvert was taken from inlet to outlet and these measurements define the X-coordinates for each acoustic anomaly. The Y-coordinates for each acoustic anomaly is defined by a height measurement from the multi plate liner section overlap that is *closest to the waterline*. The acoustic anomalies are then drawn on maps as in the example listed below.

Example of Acoustic Map Protocol:

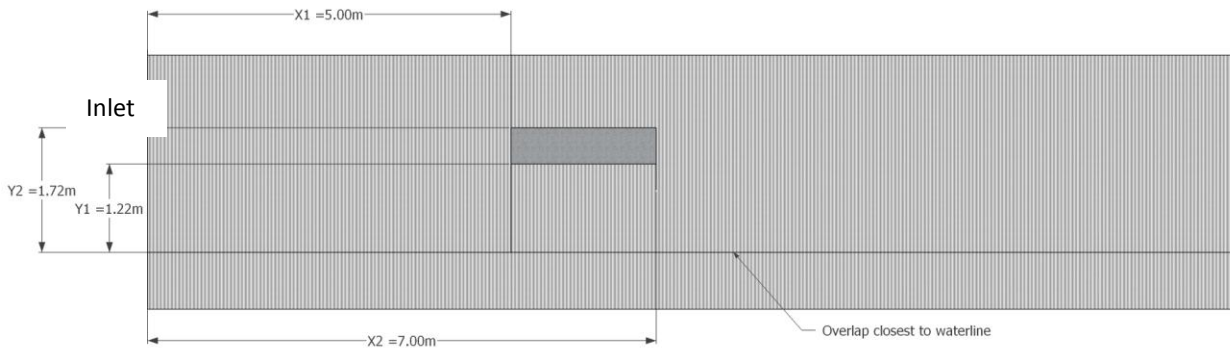
In the below example the dark grey rectangle represents an acoustic anomaly on the culvert wall. The X coordinates indicate the distance from the inlet of the culvert and the Y coordinates indicate the height from the section overlap closest to the waterline. Each corner of the rectangle is given its own pair of coordinates. Table 1 shows a sample table with reference to the example map below. For brevity, the tabular data is omitted from this report and only the acoustic maps are presented.

The purpose of this level of detail is to identify the exact location of any anomaly for future follow-up inspections and deterioration monitoring.

Table 1 example of measurement data presentation

Acoustic Anomaly #	X ₁ , X ₂ (m)	Y ₁ , Y ₂ (m)
1	5.00, 7.00	1.22, 1.72

BCT Inspection – information from the visual and acoustic inspections of an area are then selected for diagnostic BCT imaging. The BCT scanner is positioned against the culvert wall, and a region of up to 30 cm per position is imaged and assessed in real time. Depth of penetration is variable with a typical target depth of 10-15cm.



City of Toronto engineers selected six potential culverts for inspection, of these three were chosen for BCT inspection. The following are the results of the culvert inspections.

INSPECTION RESULTS – 405 LESLIE STREET

Location – Leslie Street approximately 800m north of Eglinton Avenue.

Date of Inspection – 19/09/2011

Table 2 summarizes bridge data as provided by the City of Toronto for reference.

Table 2 Bridge Data for culvert 405

No	D	Name_1	Type_1	ou	Type_2	Name_2	Location	Year
405	C	Leslie St	RD/	o	WAT	Stream	N of Eglinton	1960

Visual Inspection

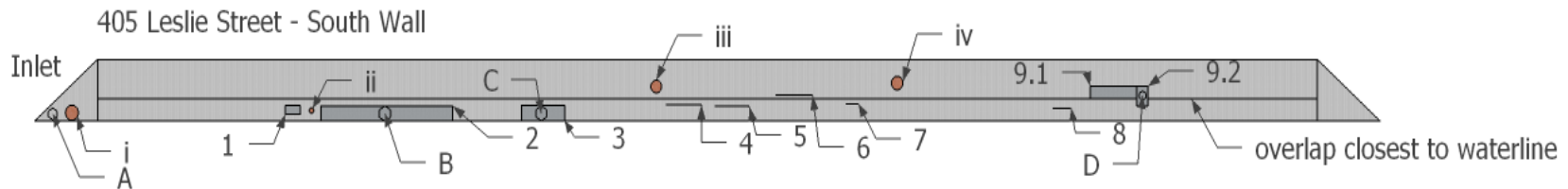
The visual inspection general appraisal for culvert 405 indicated overall poor condition – advanced section loss at inlet and outlet. Deterioration throughout the pipe with through wall holes on the upper 45 of arch at mid span was observed.

Summary

The inspection of culvert 405 – Leslie Street was conducted to evaluate the additional information available through the use of BCT to aid in the remediation/replacement decisions. The acoustic inspection mapped 22 anomalies, of these eight locations were selected for BCT imaging. Based upon the 12 BCT images it was discovered that, in the tested locations, the voids were small and shallow even though the acoustic map indicated undermining. Culvert 405 – Leslie Street showed only minor void pockets forming in the apex of several corrugations. No significant undermining was found in the imaged locations. Image C, a small void, is the most significant instance of undermining, despite 22 acoustic anomalies. This reinforces the inaccuracy of using acoustic inspection.

The results of the inspection of culvert 405 – Leslie Street follow.

Acoustic Inspection Maps – 405 Leslie Street South Wall



Legend

Acoustic Anomalies

Labeled numerically 1 through 9.2

BCT Images

A - BCT Image A - Verification image taken of known void at inlet

B - BCT Image B, C, D and E taken at this location

C - BCT Image F and G taken from this location

D - BCT Image H and I taken at this location

Selected images are shown in this report to demonstrate capabilities of BCT

Visual Indicators

i - Through wall corrosion noted on invert at inlet

ii - Corrosion

iii - Through wall corrosion noted on upper 45 of culvert barrel

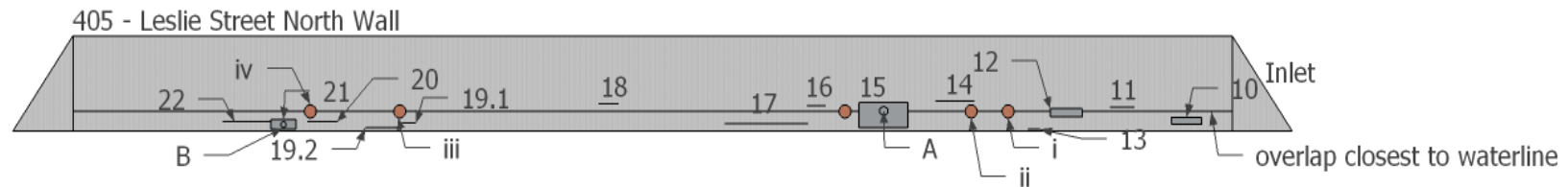
iv - Corrosion

Acoustic Anomaly: Acoustic Anomalies are assigned a numerical number, given from inlet to outlet, X position and Y position information is given, so they can be relocated in the future. In the case of large voids they are considered to be rectangles (2 X positions, 2 Y positions) and small voids considered to be a horizontal line (2 X Positions, 1 Y position)

BCT Images: Are assigned a letter with 2 X-Positions, 1 Y-position noted with description. These are designated with grey circles.

Visual Indicator: Visual indicators are assigned a roman numeral and the X position from inlet noted as well as a description. These are designated as red circles.

Acoustic Inspection Maps – 405 Leslie Street North Wall



Legend

Acoustic Anomalies

Labeled numerically 10 through 22

BCT Images

A - BCT Image J and K taken at this location

B - BCT Image L taken at this location

Selected images are shown in this report to demonstrate capabilities of BCT

Visual Indicators

i - Repair work noted at this location

ii - Repair work noted at this location

iii - Repair work and corrosion noted at this location

iv - Repair work and corrosion noted at this location

Acoustic Anomaly: Acoustic Anomalies are assigned a numerical number, given from inlet to outlet, X position and Y position information is given, so they can be relocated in the future. In the case of large voids they are considered to be rectangles (2 X positions, 2 Y positions) and small voids considered to be a horizontal line (2 X Positions, 1 Y position)

BCT Images: Are assigned a letter with 2 X-Positions, 1 Y-position noted with description. These are designated with grey circles.

Visual Indicator: Visual indicators are assigned a roman numeral and the X position from inlet noted as well as a description. These are designated as red circles.

BCT Diagnostic Images

Images

BCT images provide a cross-sectional view behind the culvert wall. In the images below the Y intercept indicates the front (accessible) side of the culvert wall. An increase in Y indicates the depth behind the wall. The X axis is the width of the BCT cross-sectional image. Note that images are best viewed on a flat panel monitor. Printed media may have reduced contrast.

405 – Leslie Street South Wall

Image A – This image was taken for verification. The image was taken of a clearly visible void section at the inlet end of the culvert on the south wall. The corrugated steel wall is visible at the bottom of the image. The dark black from approximately 12 cm to 22cm on the vertical axis indicates void. The grey that appears from approximately 22cm to 30cm on the vertical axis indicates soil.

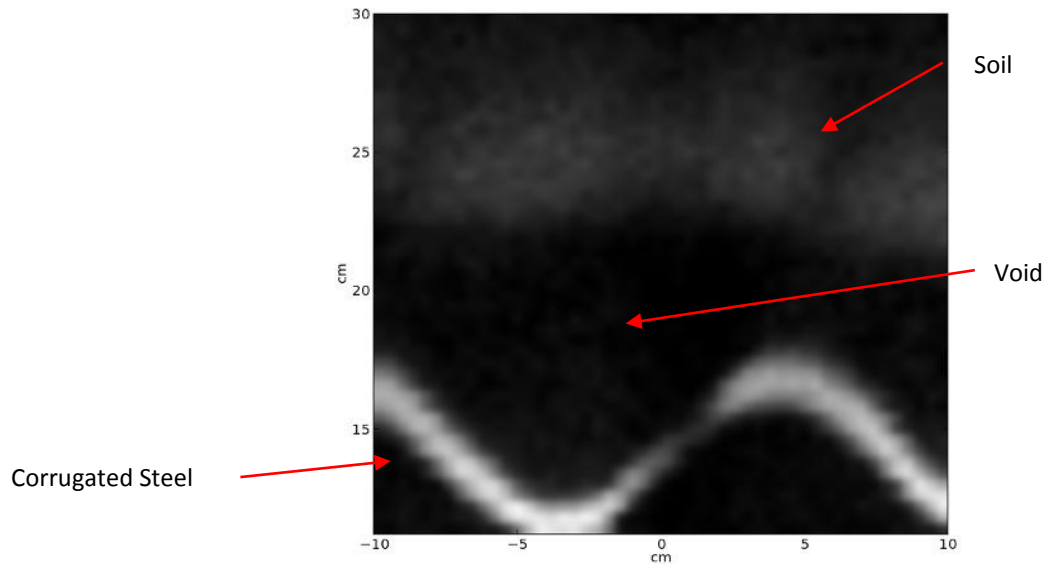


Image B – The acoustic inspection indicated audible void at Acoustic Anomaly #2, however, a follow-up BCT inspection shows supporting soil behind the culvert wall. Three images were taken along the length of Acoustic Anomaly #2 and all revealed supporting soil directly behind the culvert wall. This demonstrates the potential misinformation provided by acoustic inspections without follow up verification. Likely the acoustic variability can be attributed to a thin area along the steel wall in which soil compaction is less, therefore giving a different acoustic signal than surrounding areas.

Location: Acoustic Anomaly #2 (X = 1149cm – 1169cm, Y = -44.5cm)

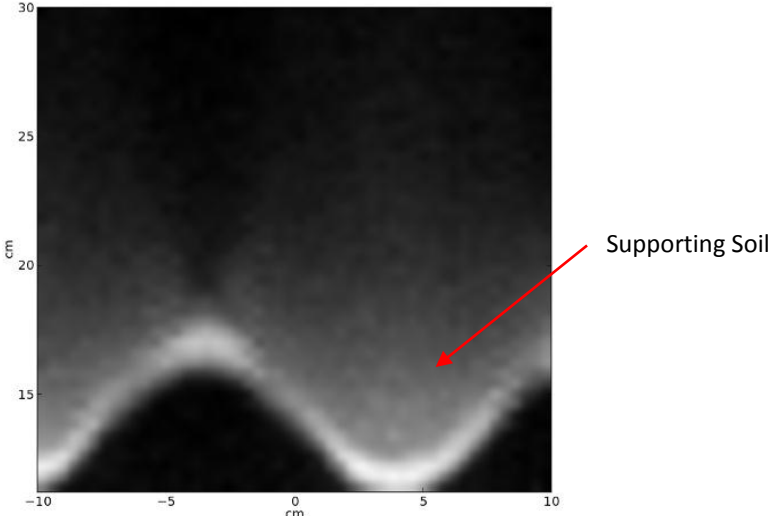
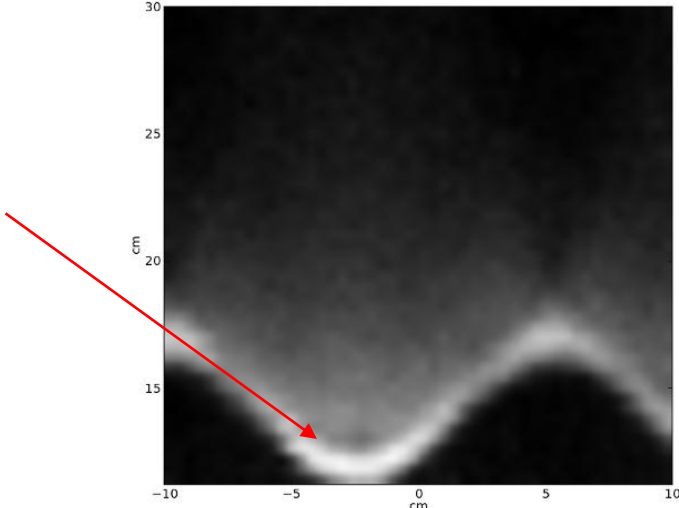


Image C – The BCT inspection of Acoustic Anomaly #3 shows a small void pocket or possibly loosely compacted material behind the culvert wall between X = -5 and X = 0.

Location: Acoustic Anomaly #3 (X = 2128cm – 2148cm, Y = -52.5cm)

Small void pocket,
magnified in Image E



405 – Leslie Street South Wall

Image D – The BCT inspection of Acoustic Anomaly #15 indicates a void pocket in the valley of the corrugation.

Location: Acoustic Anomaly #15 (X = 2145cm – 2165cm, Y = -28cm)

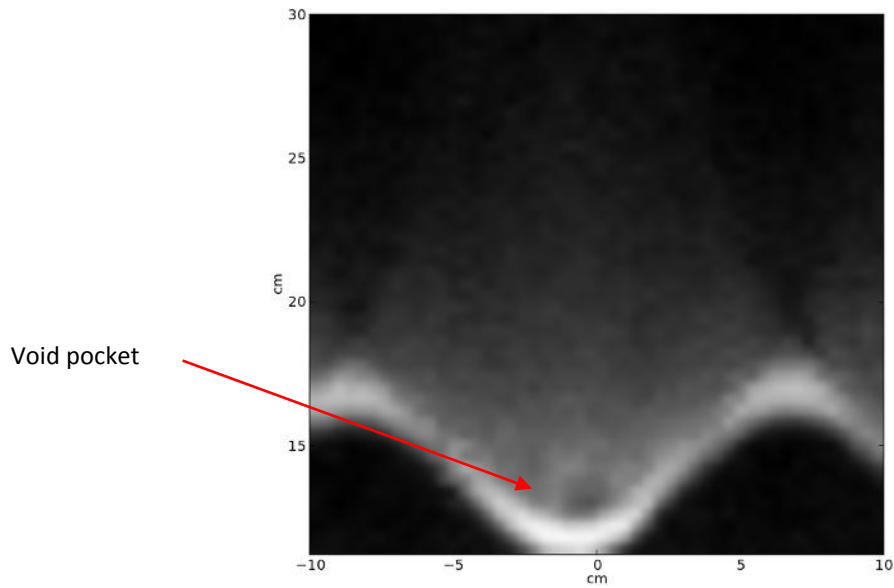
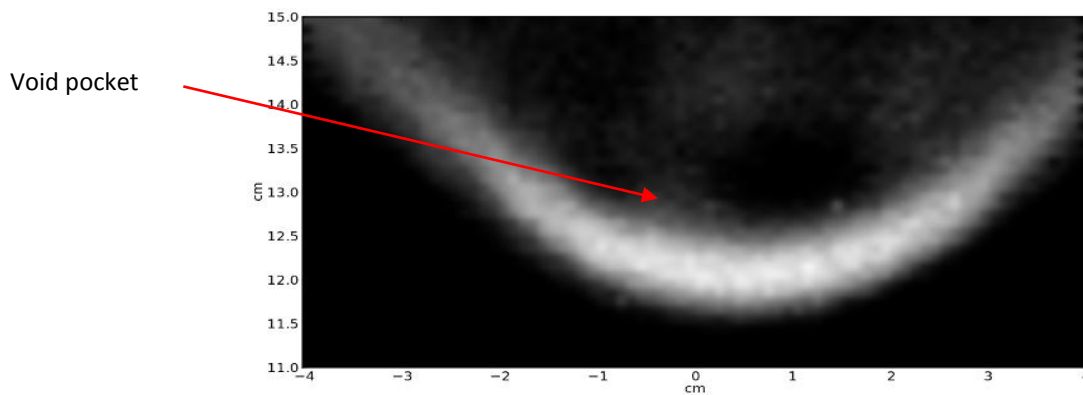


Image E – A magnified image of Acoustic Anomaly #15 in the area of concern indicates a small amount of void beginning to form in the valley of the corrugation.

Location: Close up of above – Acoustic Anomaly #15 (X = 2151cm – 2159cm, Y = -28cm)



INSPECTION RESULTS – 267 ALBION ROAD

Location – Albion Road west of Islington Avenue.

Date of Inspection – 20/09/2011

Table 3 summarizes bridge data as provided by the City of Toronto for reference.

Table 3 Bridge Data for culvert 267

No	D	Name_1	Type_1	ou	Type_2	Name_2	Location	Year
267	C	Albion Road	RD2/	o	WAT	Albion Creek	W of Islington	1964

Visual Inspection

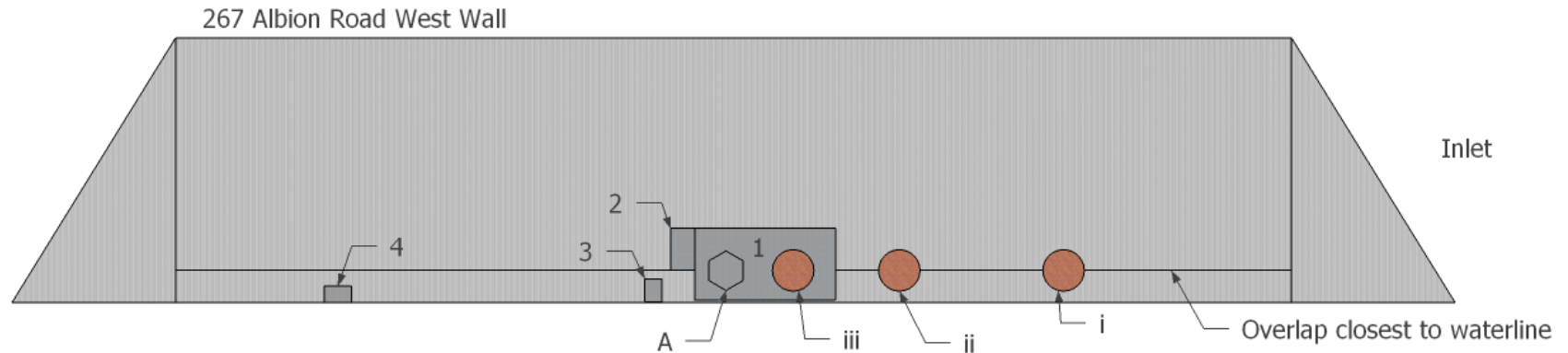
The visual inspection general appraisal for culvert 267 – Albion Road indicated satisfactory condition – structural elements show some deterioration.

Summary

The inspection of culvert 267 – Albion Road noted six acoustic anomalies. BCT images were taken at six locations. It was determined by BCT imaging that acoustic anomaly #1 had significant voiding behind the culvert wall (see Image A). In some areas there was no supporting soil for up to 15 cm behind the culvert wall. Although the voiding was significant it was limited to isolated areas within the culvert. BCT images A, B, and C of culvert 267 – Albion Road show significant undermining at acoustic anomaly location #1. Further images taken at acoustic anomaly #6 show pockets forming in the apex of the corrugation.

The results of the inspection of culvert 267 – Albion Road follow.

Acoustic Inspection Maps – 267 Albion Road West Wall



Legend

Acoustic Anomalies

Labeled numerically 1 through 4

BCT Images

A - BCT Image A taken at this location

Selected images are shown in this report to demonstrate capabilities of BCT

Visual Indicators

i - Corrosion on roof

ii - Heavy corrosion at drain

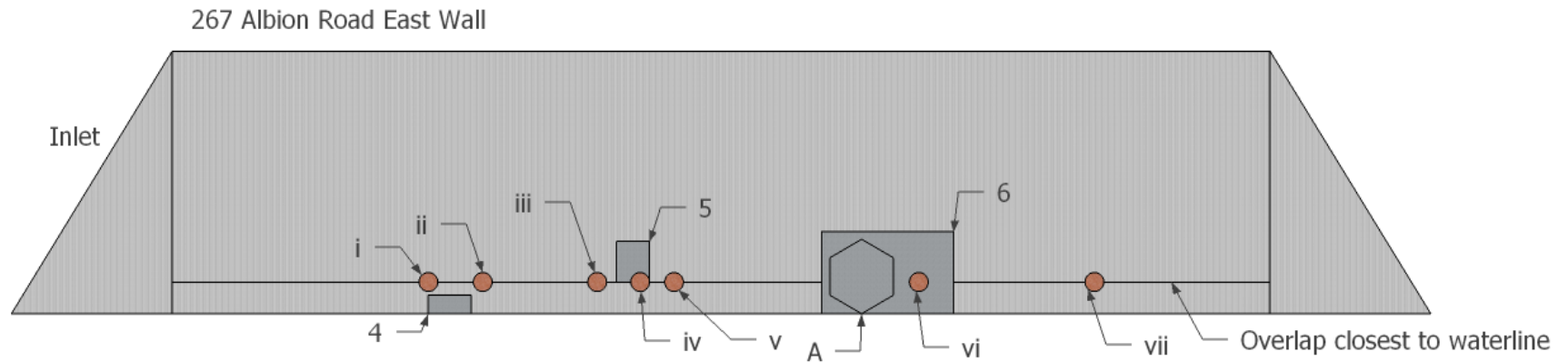
iii - Through wall corrosion

Acoustic Anomaly: Acoustic Anomalies are assigned a numerical number, given from inlet to outlet, X position and Y position information is given, so they can be relocated in the future. In the case of large voids they are considered to be rectangles (2 X positions, 2 Y positions) and small voids considered to be a horizontal line (2 X Positions, 1 Y position)

BCT Images: Are assigned a letter with 2 X-Positions, 1 Y-position noted with description. These are designated with grey circles.

Visual Indicator: Visual indicators are assigned a roman numeral and the X position from inlet noted as well as a description. These are designated as red circles.

Acoustic Inspection Maps – 267 Albion Road East Wall



Legend

Acoustic Anomalies

Labeled numerically 4 through 6

BCT Images

A - BCT Image location for images D, E and F

Selected images are shown in this report to demonstrate capabilities of BCT

Visual Indicators

- i - Missing nut
- ii - Missing nut
- iii - Missing bolt
- iv - Missing nut
- v - Heavy corrosion and missing nut and bolt
- vi - Heavy corrosion around drain
- vii - Missing nut

Acoustic Anomaly: Acoustic Anomalies are assigned a numerical number, given from inlet to outlet, X position and Y position information is given, so they can be relocated in the future. In the case of large voids they are considered to be rectangles (2 X positions, 2 Y positions) and small voids considered to be a horizontal line (2 X Positions, 1 Y position)

BCT Images: Are assigned a letter with 2 X-Positions, 1 Y-position noted with description. These are designated with grey circles.

Visual Indicator: Visual indicators are assigned a roman numeral and the X position from inlet noted as well as a description. These are designated as red circles.

BCT Diagnostic Imaging

267 – Albion Road West Wall

Image A – BCT inspection of Acoustic Anomaly #1 shows significant void behind the culvert wall.

Location: Acoustic Anomaly #1 (X = 1655cm – 1675cm, Y = 25cm)

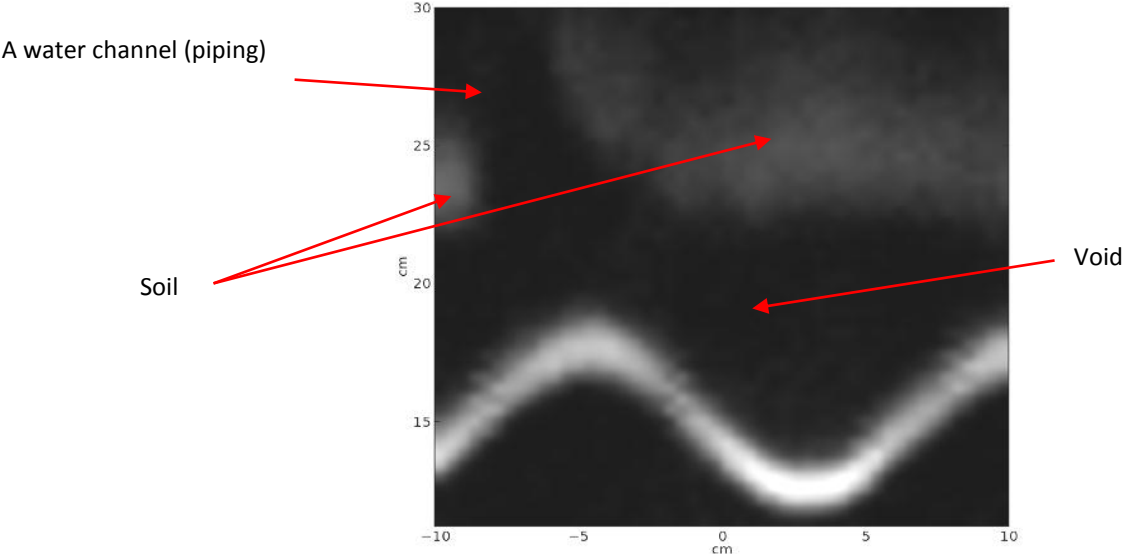


Image B – A follow up BCT image of Acoustic Anomaly #1 indicates significant (up to 15cm) voiding behind the culvert wall.

Location: Acoustic Anomaly #1 (X = 1539cm – 1559cm, Y = 17cm)

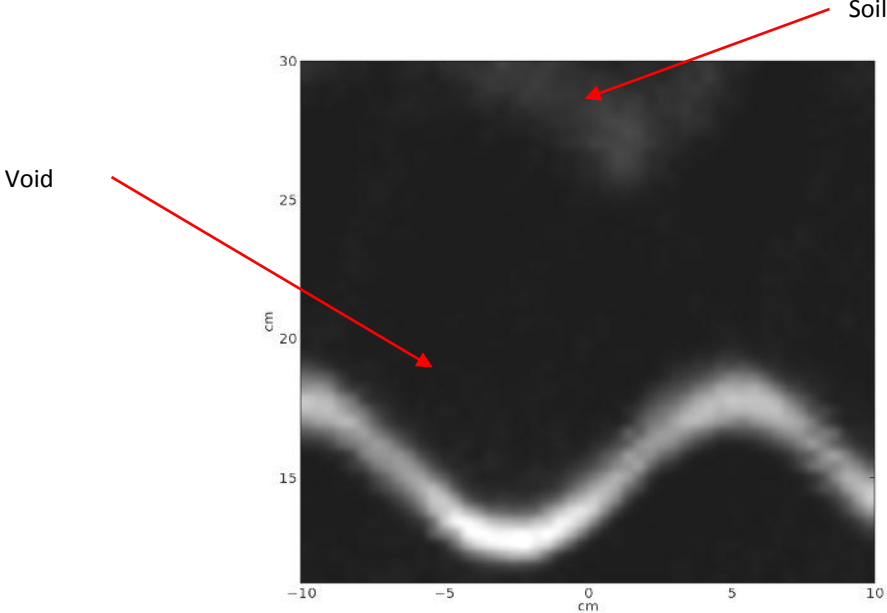
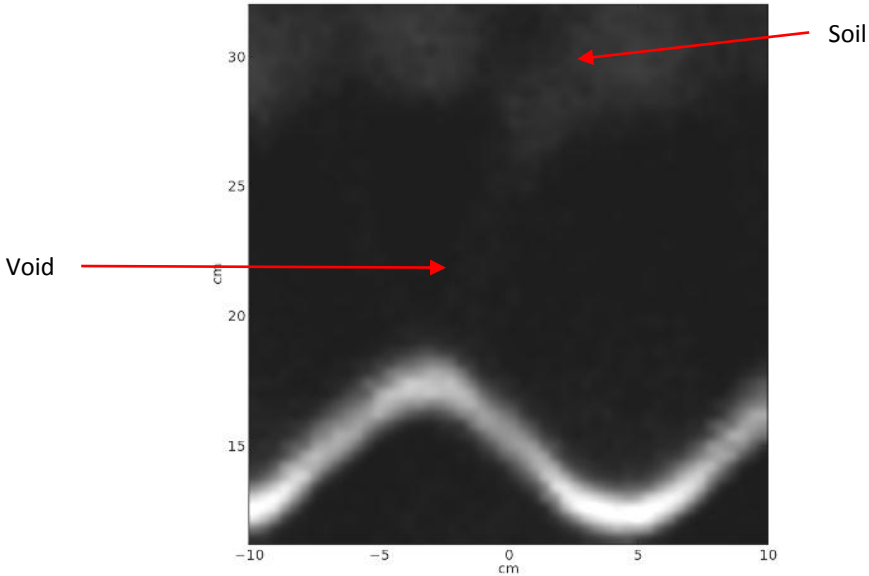


Image C - A follow up BCT image of Acoustic Anomaly #1 indicates significant (up to 15cm) voiding behind the culvert wall. Although difficult to see soil begins to appear at approximately 27cm on the vertical axis.

Location: Acoustic Anomaly #1 (X = 1456cm – 1476cm, Y = 84.5cm)



267 – Albion Road East Wall

Image D – The BCT inspection of Acoustic Anomaly #6 indicates the presence of soil directly behind the culvert wall.

Location: Acoustic Anomaly #4 (X = 1632cm – 1652cm, Y = -63.5cm)

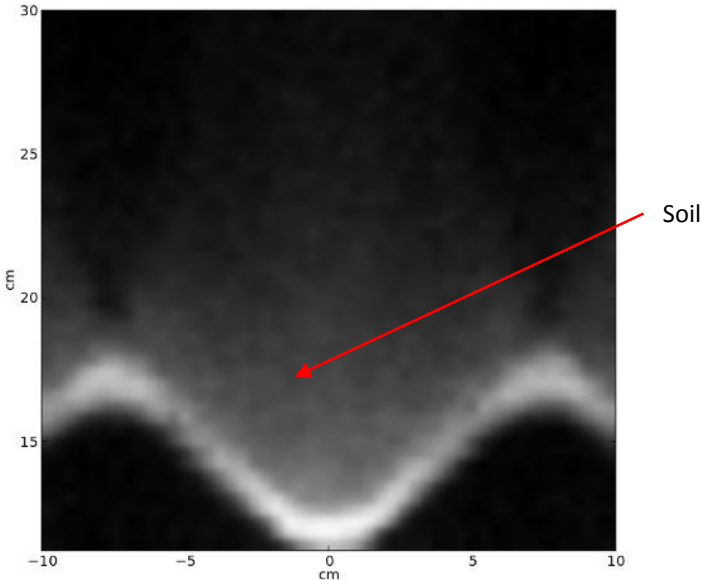


Image E – Further BCT imaging of Acoustic Anomaly #6 indicates some void in the valley of the corrugation.

Location: Acoustic Anomaly #4 (X = 1712cm – 1732cm, Y = 37cm)

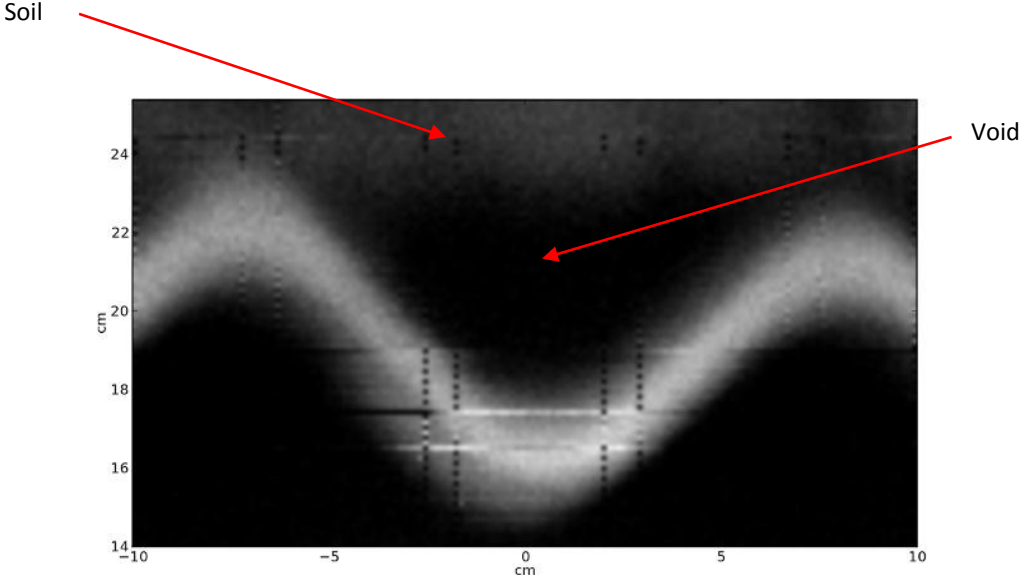
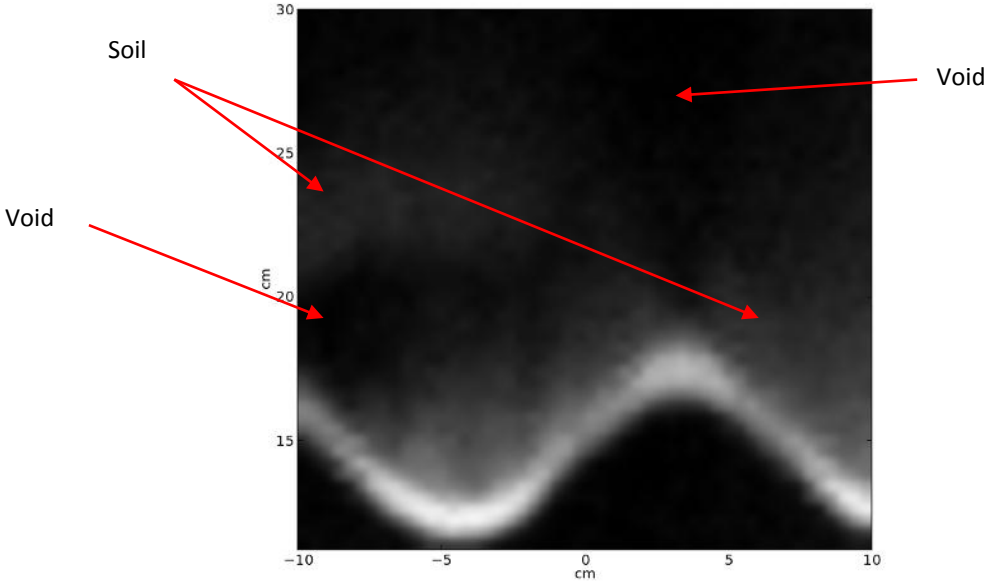


Image F – An additional BCT image of Acoustic Anomaly #6 indicates void pockets behind the culvert wall.

Location: Acoustic Anomaly #4 (X = 1897cm – 1917cm, Y = 31.5cm)



INSPECTION RESULTS – 407 ELLESMERE ROAD

Location – Ellesmere Road west of Morningside

Date of Inspection – 21/09/2011

Table 4 summarizes bridge data as provided by the City of Toronto for reference.

Table4 Bridge Data for culvert 407

No	D	Name_1	Type_1	ou	Type_2	Name_2	Location	Year
407	C	Ellesmere Road	RD/	o	WAT	Creek	W of M'Side	1980

Visual Inspection

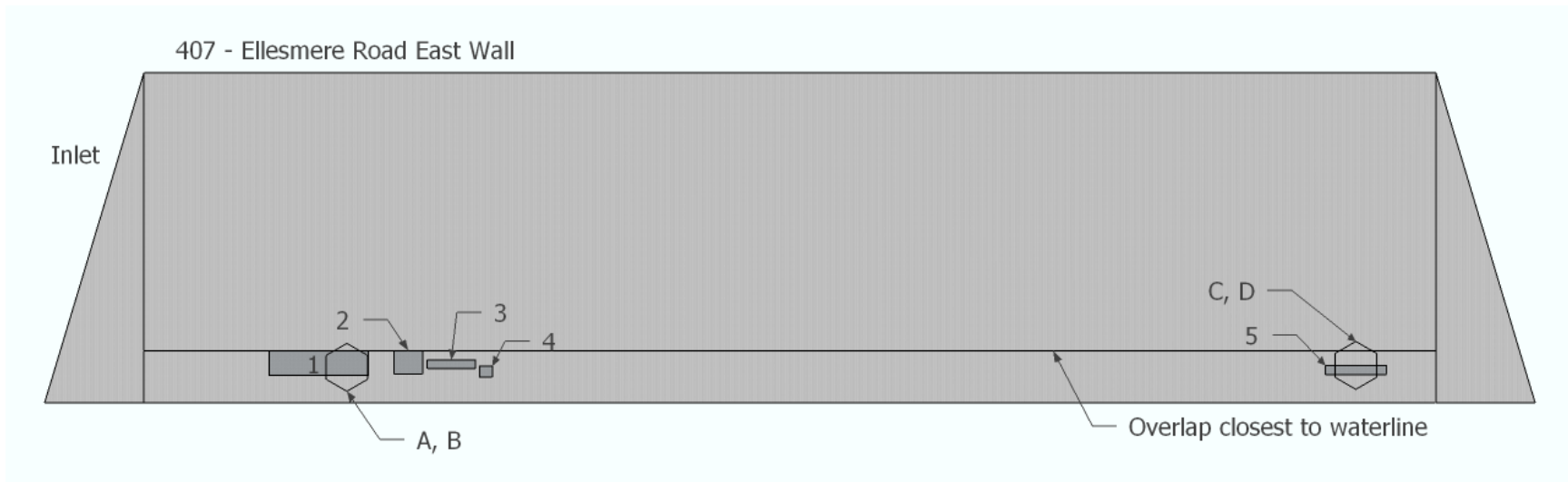
The visual inspection general appraisal for culvert 407 indicated some minor problems. The visual inspection found minimal damage resulting from drainage, vegetation, and early development of scour holes.

Summary

The acoustic inspection of culvert 407 - Ellesmere Road mapped eight acoustic anomalies. The BCT inspection imaged six locations with one region indicating a low density region indicative of loosely compacted soil or inconsistent backfill. A follow-up image in the area of concern indicated this was an isolated issue. BCT inspection of culvert 407 – Ellesmere Road indicated no major undermining present in the imaged locations.

The results of the inspection of culvert 407 – Ellesmere Road follow.

Acoustic Inspection Maps – 407 Ellesmere Road East Wall



Legend

Acoustic Anomalies

Labeled numerically 1 through 5

BCT Images

A - BCT Image location

B - BCT Image location

C - BCT Image location

D - BCT Image location

Selected images are shown in this report to demonstrate capabilities of BCT

Acoustic Anomaly: Acoustic Anomalies are assigned a numerical number, given from inlet to outlet, X position and Y position information is given, so they can be relocated in the future. In the case of large voids they are considered to be rectangles (2 X positions, 2 Y positions) and small voids considered to be a horizontal line (2 X Positions, 1 Y position)

BCT Images: Are assigned a letter with 2 X-Positions, 1 Y-position noted with description. These are designated with grey circles.

Visual Indicator: Visual indicators are assigned a roman numeral and the X position from inlet noted as well as a description. These are designated as red circles.

Acoustic Inspection Maps – 407 Ellesmere Road West Wall



Legend

Acoustic Anomalies

Labeled numerically 6 through 8

BCT Images

E - BCT Image location

F - BCT Image location

Selected images are shown in this report to demonstrate capabilities of BCT

Acoustic Anomaly: Acoustic Anomalies are assigned a numerical number, given from inlet to outlet, X position and Y position information is given, so they can be relocated in the future. In the case of large voids they are considered to be rectangles (2 X positions, 2 Y positions) and small voids considered to be a horizontal line (2 X Positions, 1 Y position)

BCT Images: Are assigned a letter with 2 X-Positions, 1 Y-position noted with description. These are designated with grey circles.

Visual Indicator: Visual indicators are assigned a roman numeral and the X position from inlet noted as well as a description. These are designated as red circles.

BCT Diagnostic Imaging

407 - Ellesmere Road East Wall

Image A – A BCT image of Acoustic Anomaly #1 confirms the presence of soil directly behind the culvert wall.

Location: Acoustic Anomaly #1 Shot_2 (X = 922cm – 942cm, Y = -99.5cm)

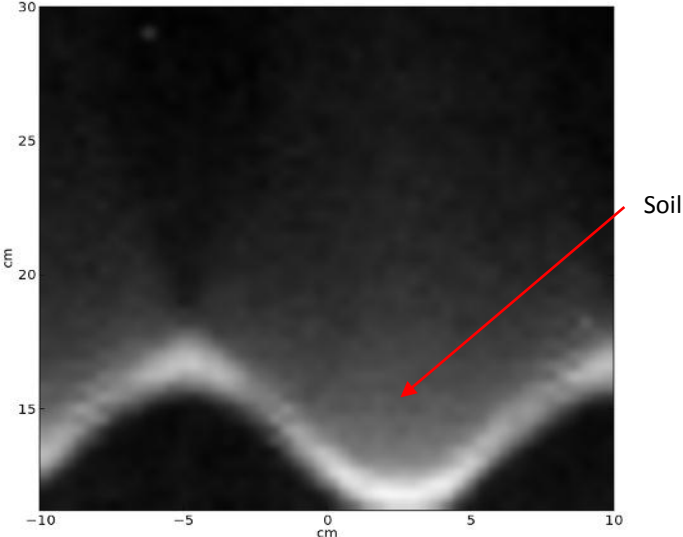
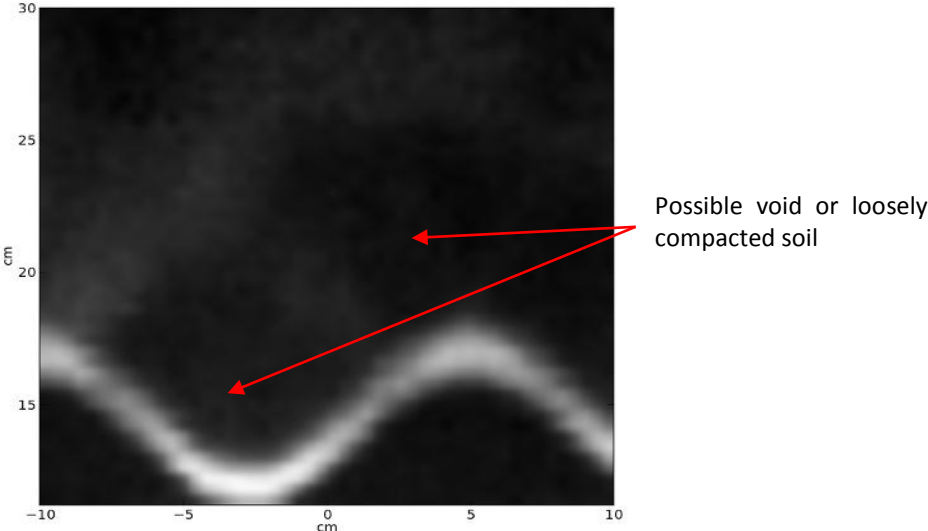


Image B – The BCT inspection of Acoustic Anomaly #5 indicates possible void or loosely compacted soil behind the culvert wall.

Location: Acoustic Anomaly #5 (X = 6502cm – 6522cm, Y = -88.5cm)



Conclusion

The primary conclusion from the report is that acoustic mapping or a “knock test” is inconsistent in identifying undermining. Although a fast and inexpensive screening tool, it falsely identifies undermining. This can lead to replacing a culvert earlier than required when minor repairs may be sufficient. Consequently, a confirmation tool such as BCT is needed to conclusively measure undermining.

The demonstrated pipe inspection protocol employing BCT provides information for the following issues:

1. Quantifying necessary repairs
2. Detecting nonconformance early
3. Trending deterioration – repair when necessary
4. Repairing voids prior to slip-lining
5. Optimizing asset lifespan with proper and necessary maintenance

Insight was also gained in using modern diagnostic imaging tools to understand anomalies. In particular, the piping visible in Image A on the west wall of Albion Road is enlightening. The location and direction give indications of the possible cause of undermining to assist with preventive repairs. A storm drain is near this location and is likely the culprit. Much like in medicine, CT imaging provides meaning that enables diagnostic analysis. With clear visualization of the piping, rehabilitation can be more effectively planned. For example, repair work can be conducted to fill the undermining, as well as repair the adjacent storm drain. Follow-up BCT images can be taken over time to test if the rehabilitation is successful. Again analogous to medicine, with the proper “diagnosis”, one can “save the patient”; the culvert can be repaired avoiding the much

higher cost of asset failure and then total replacement. In the trial, BCT quantified undermining of major roadway culverts, which is essential information in planning replacement and rehabilitation.

References

- [1] Lale, P.G., *The examination of internal tissues, using gamma-ray scatter with a possible extension to megavoltage radiography*, Phys. Med. Biol., vol. 4, pp. 159-167, 1959.
- [2] Clark, R.L. and Van Dyk, G., *Compton-scattered gamma rays in diagnostic radiography*, Medical Radioisotope Scintigraphy, STI/PUB/193 Vienna: IAEA, pp. 247.
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